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A watt balance based on a simultaneous measurement scheme

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Abstract
The International Bureau of Weights and Measures (BIPM) is developing a novel watt balance based on a simultaneous measurement scheme for the forthcoming redefinition of the kilogram. The two distinct measurement phases in a conventional watt balance are carried out in a single phase where all quantities are measured simultaneously. The main characteristics of this simultaneous measurement approach are described. An analysis of the advantages and the drawbacks is carried out.

Keywords: watt balance, mass unit, Planck constant

(Some figures may appear in colour only in the online journal)

1. Introduction

The BIPM is the custodian of the international prototype of the kilogram, which defines the mass unit according to the current definition of the kilogram. The BIPM is developing a novel watt balance in order to realize the forthcoming new definition of the kilogram in terms of the Planck constant, h, at the 10^{-8} level [1]. At present, however, the watt balance is used to measure h in terms of a test mass that is traceable to the mass of the international prototype. The main distinguishing feature of the BIPM watt balance is its capability to carry out a simultaneous measurement scheme, in addition to the conventional two-phase scheme. Several aspects of this novel measurement approach have been treated elsewhere but only partially [2, 3]. This paper aims to give a more general and a more complete analysis of a one-phase watt balance like the one under development at the BIPM.

2. Principle

A watt balance makes use of two physical laws that are based on the interaction between a magnetic field $\vec{B}$ and an inductive coil. When a current I is driven through the coil, an electromagnetic force is exerted on each infinitesimal segment of wire $\mathrm{d}\vec{L}$ according to the Lorentz law:

$$\mathrm{d}\vec{F} = I \cdot (\mathrm{d}\vec{L} \times \vec{B}). \tag{1}$$

When the coil moves at a velocity $\vec{v}$ through the magnetic field, an electromotive force is induced across the segment according to Faraday's law:

$$\mathrm{d}U = \vec{v} \cdot (\mathrm{d}\vec{L} \times \vec{B}). \tag{2}$$

These equations are combined to cancel the $(\mathrm{d}\vec{L} \times \vec{B})$ term:

$$\mathrm{d}U \cdot I = \mathrm{d}\vec{F} \cdot \vec{v}. \tag{3}$$

After integration over turns of the coil, (3) becomes

$$U \cdot I = \vec{F} \cdot \vec{v} + \vec{\tau} \cdot \vec{\omega}, \tag{4}$$

where an additional term appears due to a possible angular velocity $\vec{\omega}$ of the coil and a non-symmetric Lorentz force along the coil which produces a torque $\vec{\tau}$ [4]. The vertical component of the Lorentz force is used to balance the weight of a test mass m subjected to the gravitational acceleration g. The direction of g defines the vertical, or z, axis. Equation (4) can be rewritten as

$$U \cdot I = mgv_z\left[1 + \left(\frac{F_x}{F_z} \cdot \frac{v_x}{v_z} + \frac{F_y}{F_z} \cdot \frac{v_y}{v_z}\right) + \left(\frac{\tau_x}{F_z} \cdot \frac{\omega_x}{v_z} + \frac{\tau_y}{F_z} \cdot \frac{\omega_y}{v_z} + \frac{\tau_z}{F_z} \cdot \frac{\omega_z}{v_z}\right)\right], \tag{5}$$

where the indices x and y represent the horizontal components. In the watt balance, only the vertical force and the velocity are accurately determined. The watt balance should thus be aligned in order to minimize the non-vertical terms in (5). If the Lorentz force and the velocity of the coil are solely vertical along the g vector, then the equation establishes the equivalence of the virtual electrical power UI and mechanical power mgv_z. That means the equality of the F/I and U/v ratios, each of which measures the BL term where B is the magnetic flux density and L the wire length of the coil. Traceability of the electrical quantities to the macroscopic quantum electrical standards provides the link between the mass m and h. The ingenuity of the watt balance concept is that direct determinations of B and L are not required and that the virtual electrical and mechanical powers can be compared so that parasitic energy losses, such as for example due to friction, do not lead to measurement errors.

Measurements of the F/I and U/v ratios can be carried out separately or simultaneously. Most existing watt balances follow the method first proposed by Kibble [5], in which the measurements are performed in two distinct phases [6]. In this measurement scheme, each quantity can be measured in one of the phases without any influence from the other phase. However, there are several disadvantages. Equation (3) is exact only if the term $(\mathrm{d}\vec{L} \times \vec{B})$ remains constant during the two measurement phases, which is not necessarily true in practice. First, it is possible that drift over time may occur. The magnetic flux density and coil length vary as a function of environmental temperature as characterized by their temperature coefficients. Variations in external magnetic sources also influence the constancy of the total magnetic flux density. The current flowing during the force measurement introduces not only an additional magnetic field but also a temperature increase due to resistive heating. Secondly, the coil position inside the magnetic field is particularly sensitive to a change during the transition between the two measurement phases. Since the magnetic field is neither perfectly uniform along the vertical position nor perfectly radially symmetrical, any change in the coil position leads to inequality of the two BL products. Finally, the BL term is evaluated from static measurements at one central position in the force phase while it is deduced from dynamic measurements along the magnetic field in the velocity phase. The BL product at the central position needs to be perfectly coherent with data taken from the velocity phase.

Although these disadvantages are manageable and result in small effects in several existing two-phase watt balances, they can be overcome in the simultaneous measurement scheme. The two ratios F/I and U/v, measured at the same time and using the same experimental set-up, sense exactly the same BL product, i.e. the coil with the same alignment occupies the same place inside the same magnetic field. The comparison of the two types of powers is thus exact and the two additional terms in (5) are exact error terms which can be corrected. Conversely, any imperfect alignment of the watt balance will inevitably cause the BL product to change during the two-phase scheme. These misalignment terms may be difficult to correct in practice and are often dealt with as uncertainty components. Although the apparatus still needs to be correctly aligned to avoid applying corrections, the simultaneous measurement scheme is less sensitive to changes in magnetic field and coil alignment compared with the two-phase scheme. In addition, comparison of the virtual electrical and mechanical powers is realized all along the travel length inside the magnetic field. This means that the Planck constant is determined at a large number of positions rather than at one single position as in a two-phase watt balance. Weighing at different positions along the moving trajectory is also possible in a two-phase balance but this would be time consuming. The simultaneous scheme allows more h data to be obtained from one complete measurement cycle. Moreover, the h value is independent of the position inside the magnetic field so that the determined experimental data should be randomly distributed. Any position dependence should indicate some systematic measurement or alignment errors. This will help one to check and confirm the predicted operation of the experiment. The drawback of the simultaneous scheme is the undesirable influence of the measurements of one ratio on the simultaneous measurement of the other.

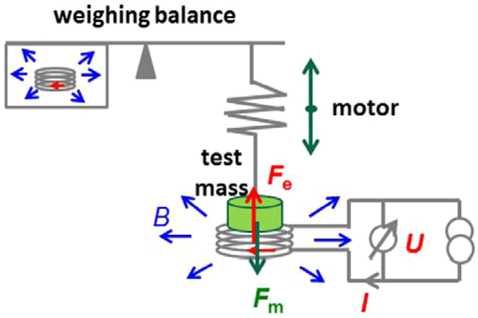


Figure 1. Schematic drawing of a watt balance operating in the simultaneous measurement scheme. The weight of the test mass F_m is balanced by the electromagnetic force exerted on the moving coil F_e.

3. Energy losses

As mentioned above, the conventional two-phase watt balance establishes the equivalence of the *virtual* electrical and mechanical powers $UI = mgv$ because the induced voltage U and the current I are not present simultaneously, and the velocity v is measured while the mass m is not on the weighing plateau. Therefore, any dissipative energy loss, e.g. due to friction, does not enter into the basic equation of the experiment. For a one-phase watt balance, induced voltage and current are present at the same time, and the mass is loaded while the coil is moving, and the question arises whether any dissipative energy losses influence the measurement equation. Figure 1 shows the schematic set-up of a one-phase watt balance, similar to the present BIPM device. The suspension, including the motor and the coil, hangs from the weighing balance.

The current flow through the coil is chosen so that the weight of the test mass is equal to the Lorentz force on the coil $mg = IBL$. Any imbalance between the two forces is detected by the weighing balance which is kept in dynamic equilibrium by a servo-control loop. If some friction or energy dissipation occurs in the system, the motor has to provide a larger force to maintain the velocity. The motor has its separate power supply and its additional effort does not influence the coil current supplied by the constant current source. If the friction occurs within the suspension, the reading of the weighing balance is not influenced by it. If the friction occurs between the suspension and an external element, not suspended from the weighing balance, this is seen as an additional force. If this force is the same during the measurements with and without the mass, it has the same effect as a small additional constant mass on the suspension and is eliminated by combining the results of these two measurements. Similarly, if an additional force acting on the masses (e.g. eddy current damping due to a conductive mass moving in a non-uniform magnetic field) is the same at constant position during the coil upward and downward movements, its effect is removed by averaging the results of the measurements in opposite velocities.

The voltage induced across the coil is proportional to the velocity generated by the motor $U = vBL$. If there is some friction or other dissipation in the system, as said above, the motor has to provide a larger force to maintain the same velocity. The induced voltage will remain exactly the same. If the velocity were to drop somewhat due to the friction, the induced voltage would be reduced by the same proportion, but since both are measured synchronously, the above equation still holds.

These two arguments show that although dissipative processes will always be present, the additional energy is provided by the motor and does not need to be considered via additional terms in the measurement equation. This is a fundamental difference from the technique of magnetic levitation, in which the electromagnetic energy of a superconducting coil–magnet assembly served directly to increase the potential energy of a floating body. Dissipative energy losses influenced this energy conversion scheme directly and were the main reason to abandon this technique.

4. Apparatus design

The particularity of the design of a one-phase watt balance lies in the capability to carry out the two measurement phases, the weighing and the moving phase, simultaneously. In principle, a one-phase watt balance can also operate in the conventional two-phase scheme without any significant degradation in the measurement accuracy whereas the contrary is generally not the case. In a two-phase watt balance, devices and instruments are designed to correctly operate in their relevant phase but not in the other. A one-phase watt balance, such as the BIPM apparatus, thus has additional advantages. Measurements could be successively carried out in the one-phase and two-phase measurement schemes using the same apparatus and within the same series of measurement, i.e. the same measurement configuration and condition. Comparison of the results obtained using the two schemes will provide a better understanding of the main error sources and possibly reveal some unexpected effects unique to each measurement scheme. This will help one to improve the reliability of the experiment and to increase confidence in the results.

Special attention must be paid to the mechanical design of a watt balance apparatus. In contrast to the voltage and the current, the force and the velocity are vectors so that any misalignment would lead to errors (see (5)). A high-precision weighing balance is extremely sensitive to tilting and is designed for static operation. On the other hand, a purely vertical movement of the coil, suspended from the weighing balance, with respect to the magnetic field needs to be generated by a translation device. The relative motion could conceivably be generated by displacing the magnetic flux source and fixing the weighing balance as well as the suspension, including the coil. However, the environmental magnetic field will contribute to the force measurement but not to the induced voltage and thus good magnetic shielding is required. In addition, it is challenging to move the magnet, which is usually very heavy, at a constant velocity along a uniform axis. In practice, therefore, the coil is displaced. Particular care is needed with a one-phase watt balance in order to take into account the additional constraints imposed by the simultaneous measurements. The weighing balance needs to remain static while the coil that is suspended from the balance moves. Actually, the performance of the weighing balance is very likely to be degraded by tilts introduced through motion of its support, even a smooth and well controlled motion. A translation stage integrated in the suspension between the weighing balance and the coil is thus necessary to generate the required coil motion with respect to the magnetic circuit, while leaving the balance stationary. Since the translation stage is a part of the suspension hanging from the weighing balance, its mass should be small enough in order to keep the total mass of the suspension within the measurement range of the balance with a sufficient measurement resolution. Two possible experimental set-ups are shown in figure 2.

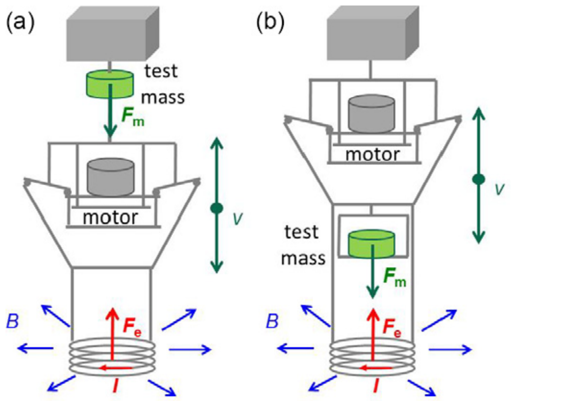


Figure 2. Experimental set-up of a simultaneous watt balance. (*a*) Symmetrical configuration where the driving motor is balanced; (*b*) unsymmetrical configuration where the driving motor is unbalanced (current BIPM configuration).

In figure 2(*a*), the test mass hangs directly from the weighing balance above the driving motor. A load is placed on the counterweight side of the motor in order to reduce the force necessary to drive the suspended coil placed on the lower part of the suspension. It uses levers to transfer the displacement of the motor to the coil. As mentioned above and explained in detail in section 5, two measurements are carried out in practice with the watt balance mass loaded or removed and the electromagnetic force pointing upwards or downwards. The electromagnetic force is set to be equal to half of the weight of the test mass so that the total force exerted on the weighing balance is the same for both measurements and within the measurement range of the weighing balance. In this configuration, the force needed to move the coil cannot be minimized. In a symmetric design, the total load on each side of the motor has the same mass and the motor is unbalanced by the electromagnetic force, which therefore needs to be overcome by the motor.

The test mass is a part of the coil side of the motor in the second configuration (figure 2(*b*)). In this case, the total force measured by the weighing balance is constant and the two sides of the motor are balanced, requiring the minimum force to drive the coil in both measurement steps. Nevertheless, the mass of the total load on the counterweight side of the motor is either larger or smaller than that on the coil side of the motor. When the coil moves upwards and downwards, the load of the counterweight side is displaced with the same absolute velocity but in the opposite direction. The total inertial force related to the loads of both sides would not be completely compensated and thereby detected by the weighing balance. In the present BIPM watt balance apparatus, an electrostatic rather than an electromagnetic motor is used to minimize unwanted magnetic forces on the test mass. As this type of motor is not strong enough to drive the coil in the first configuration (figure 2(*a*)), the configuration shown in figure 2(*b*) is used.

In addition to the specific mechanical design, the simultaneous measurement scheme requires several additional and demanding components as well as some constraints on practical realization. To dampen oscillation and vibration of the suspension, only dampers making use of the interaction among internal elements of the ensemble hung from the weighing balance are allowed. Devices based on interactions with elements external to the suspension, for example by short-circuiting the coil, cannot be used in a simultaneous watt balance. The presence of the induced voltage in the coil-current circuit requires a more demanding and careful design of the stable current source. The sign of the induced voltage reverses when the coil changes direction and this can cause small shifts on the delivered current value. Since the current and the voltage are measured simultaneously, two Josephson voltage standards are necessary rather than the one needed in a two-phase watt balance. The current in the coil is kept constant and the variations in force are measured by the weighing balance. Due to the (small) non-uniformity of the magnetic field and the relatively large time constant of a high-precision weighing balance, the velocity of the coil displacement should be small enough so that the electromagnetic force variation along the magnetic field can be tracked by the weighing balance which, as mentioned above, is designed for static operation. The acquisition of the four principal quantities, force, velocity, voltage and current, needs to be perfectly synchronized or the time constants of each measurement must be modelled. A compromise needs to be made to take into account the operation of each instrument and the noise on each measurement signal.

5. Practical measurements

As mentioned above, both the two-phase and simultaneous measurement schemes employ a differential method in order to remove constant offsets, in particular the mass of the suspension. Table 1 compares one complete measurement cycle carried out using the two-phase and the simultaneous measurement schemes. Measurement steps as well as the related measurement equations are listed. The assumptions needed to deduce the associated *BL* products and the watt balance equations are indicated. The following are the symbols and indices employed in the table:

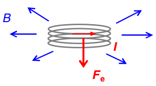
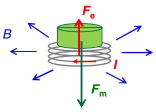
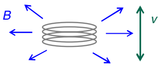
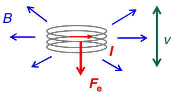
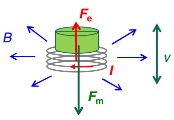
e: mass reading of the weighing balance;
F_0: difference of the total force exerted on the load side of the weighing balance, except the weight of the test mass and the Lorentz force on the coil, against that of the counterweight side;
U: voltage reading of the voltmeter used to measure the voltage across the coil;
U_0: voltage offset of the voltage measurement circuit;
F_a: force due to the accelerated masses detected by the weighing balance;
R: resistance of the coil;
$_{\mathrm{w}}$, $_{\mathrm{m}}$: weighing and moving experiments of a two-phase watt balance respectively;
$_1$, $_2$: mass unloaded and loaded, respectively;
$_{\mathrm{u}}$, $_{\mathrm{d}}$: coil moves upwards and downwards, respectively.

The upward vertical direction is defined as positive so that the inertial force due to acceleration in this direction will produce a negative signal in the weighing balance. For simplicity, the local acceleration of gravity and the absolute currents injected through the coil are assumed to be the same during the measurement cycle. The spatial gradient and the variation over the measurement time of g are not larger than 10^{-8} in practice. The current across the coil can be set and stabilized at the 10^{-8} level using a well-designed current source and the induced voltage associated with the current variation is negligible. The term due to the current effect on the magnetic field is indicated only in the two-phase measurement scheme. The two-phase scheme consists of three measurement steps and auxiliary transition phases to switch from one step to the other. The force phase is composed of two steps where the gravitational and the electromagnetic forces are compared. The polarity of the current is reversed with the mass loaded or unloaded. In the moving phase, the inductive coil moves successively upwards and downwards. The following assumptions are required to deduce the watt balance equation:

- constancy of U_0 and BL between the successive up and down coil displacements of the velocity experiment; in practice, slow drifts of U_0 and BL can be curve-fitted, leading to a slightly increased uncertainty,
- constancy of F_0 of BL between the two steps of the force experiment; in practice the effective B-field might show a small dependence on the current direction, the effect of which, however, tends to cancel out in the calculation,
- deduction of the BL value at the z_0 position from the BL profile measured in the velocity experiment,
- constancy of BL between the force and the velocity experiments; in practice, slow changes can be curve-fitted.

In the simultaneous measurement scheme, one complete cycle consists of two measurement steps with the mass loaded or

Table 1. One complete measurement cycle of a watt balance operated in the two-phase and the simultaneous measurement schemes.

Phase	Step	Measurement equations	Assumptions	*BL* products/combination	Watt equation
Two-phase measurement scheme					
Force	mass Off 	$e_{w1}g = IB_{w1}L_{w1} + F_{0w1}$ $B_{w1} = B_{0w1}[1 + f(I)]$	$F_{0w1} = F_{0w2}$ $B_{0w1}L_{w1} = B_{0w2}L_{w2}$ $f(I) + f(-I) = 0$	$B_wL_w = (mg - \Delta e_w g)/2I$ $\Delta e_w = e_{w2} - e_{w1}$	Assumption $B_wL_w = B_mL_m$ $\Downarrow$ $(mg - \Delta e_w g)/2I$ $= \Delta U_m/\Delta v_m$
Z_0	mass On 	$e_{w2}g = -IB_{w2}L_{w2} + mg + F_{0w2}$ $B_{w2} = B_{0w2}[1 + f(-I)]$			
Velocity	Up & Down 	$U_{mu} = v_{mu}B_{mu}L_{mu} + U_{0mu}$ $U_{md} = v_{md}B_{md}L_{md} + U_{0md}$	$U_{0mu} = U_{0md}$ $B_{mu}L_{md} = B_{mu}L_{md}$	$B_mL_m = \Delta U_m/\Delta v_m$ $\Delta U_m = U_{md} - U_{mu}$ $\Delta v_m = v_{md} - v_{mu}$	
Simultaneous measurement scheme					
	mass Off Up & Down 	$e_{1u}g = IB_{1u}L_{1u} + F_{01u} - F_{a1u}$ $U_{1u} = v_{1u}B_{1u}L_{1u} + U_{01u} + R_{1u}I$ $e_{1d}g = IB_{1d}L_{1d} + F_{01d} - F_{a1d}$ $U_{1d} = v_{1d}B_{1d}L_{1d} + U_{01d} - R_{1d}I$	$F_{01u} = F_{01d}$ $U_{01u} = U_{01d}$ $B_{1u}L_{1u} = B_{1d}L_{1d}$	$B_1L_1 = \left[\bar{e}_1 g - (\overline{F_{01}} - \overline{F_{a1}})\right]/I$ $\bar{e}_1$, $\overline{F_{01}}$, $\overline{F_{a1}}$: mean of up and down data $B_1L_1 = (\Delta U_1 - \Delta R_1 I)/\Delta v_1$ ΔU_1, ΔR_1, Δv_1: change between down and up data $\Rightarrow \frac{\bar{e}_1 g - (\overline{F_{01}} - \overline{F_{a1}})}{I} = \frac{\Delta U_1 - \Delta R_1 I}{\Delta v_1}$	Assumption $\overline{F_{01}} = \overline{F_{02}}$ $\Downarrow$ $(mg - \Delta\bar{e}g - \Delta\overline{F_a})/I$ $= \left(\frac{\Delta U_2}{\Delta v_2} + \frac{\Delta U_1}{\Delta v_1}\right)$ $- \left(\frac{\Delta R_2 I}{\Delta v_2} - \frac{\Delta R_1 I}{\Delta v_1}\right)$ $\Delta\bar{e} = \bar{e}_2 - \bar{e}_1$ $\Delta\overline{F_a} = (\overline{F_{a2}} - \overline{F_{a1}})$
	mass On Up & Down 	$e_{2u}g = -IB_{2u}L_{2u} + mg + F_{02u} - F_{a2u}$ $U_{2u} = v_{2u}B_{2u}L_{2u} + U_{02u} + R_{2u}I$ $e_{2d}g = -IB_{2d}L_{2d} + mg + F_{02d} - F_{a1d}$ $U_{2\text{-}d} = v_{2d}B_{2d}L_{2d} + U_{02d} - R_{2d}I$	$F_{02u} = F_{02d}$ $U_{02u} = U_{02d}$ $B_{2u}L_{2u} = B_{2d}L_{2d}$	$B_2L_2 = \left[mg - \bar{e}_2 g + (\overline{F_{02}} - \overline{F_{a2}})\right]/I$ $\bar{e}_2$, $\overline{F_{02}}$, $\overline{F_{a2}}$: mean of up and down data $B_2L_2 = (\Delta U_2 + \Delta R_2 I)/\Delta v_2$ ΔU_2, ΔR_2, Δv_2: change between down and up data $\Rightarrow \frac{mg - \bar{e}_2 g + (\overline{F_{02}} - \overline{F_{a2}})}{I} = \frac{\Delta U_2 - \Delta R_2 I}{\Delta v_2}$	

unloaded. In each step, the coil (together with or without the test mass) moves successively upwards and downwards while the forces are compared. The assumptions needed to deduce the indicated watt balance equation are the following:

- constancy of F_0, U_0 and BL between the successive upward and downward coil motions in each measurement step; in practice slow drifts of U_0 and BL can be curve-fitted,
- constancy of F_0 between the two measurement steps.

The effects due to the resistive voltage drop and the coil acceleration (section 6) are directly included in the actual watt balance equation and no assumption needs to be made.

In practice, successive up and down displacements of the coil are continuously carried out and repeated several times to reduce the type A uncertainty. These measurements are carried out over a relatively short time because they do not require any operation that might perturb the apparatus. A one up and one down sequence takes typically a few minutes with a total duration of about half an hour for ten repetitions. In the two-phase scheme, the transition from the dynamic velocity phase to the static force phase generally requires more than 10 min. This transition might introduce a significant change in the mechanical ensemble, especially the coil position depending on the apparatus design. Another transition between the two measurement steps is then needed to load or unload the test mass and to reverse the polarity of the injected current. This has a negligible influence on the mechanical ensemble if reasonable precautions are taken. This change takes about 5 min, mostly due to signal stabilization, and the force measurement takes several minutes in each step. In the BIPM one-phase watt balance, 20 min are needed to switch between the two steps. Concerning the environmental conditions, a thermal drift inside the apparatus enclosure of less than 30 mK during the duration of a set of watt balance measurements, i.e. 10 h, is a reasonable design goal. The relative thermal coefficient of the magnetic flux density $\partial B/(B\partial T)$ produced by the permanent magnets made of Sm_2Co_{17}, commonly used in the watt balances, is about -3×10^{-4} K^{-1}. The linear thermal expansion of copper wires used to make the coil in most existing watt balances is smaller and its effect is further reduced by the $1/r$ dependence of the magnetic field.

The condition of temporal stability for the voltage offset U_0 and the BL product can be easily satisfied in both measurement schemes during the whole measurement set by careful design and appropriate operation. In the simultaneous scheme, the interaction between the moving magnetic field from the coil and the source field could be a potential source of error. The more difficult assumption concerns the force offset F_0, which should be the same for each weighing measurement in the two-phase scheme and for all the measurements in the simultaneous scheme. As mentioned in section 3, the eddy-current-generated force exerted on the moving conductive masses needs to be reproducible enough in order to cancel by combining the up and down velocity measurements.

The two-phase measurement scheme requires good cancellation of the current effect on the magnetic field by averaging the results for both current directions. It requires two additional assumptions, the effects of deviations from which need to be carefully considered.

In a one-phase watt balance, the actual equation establishing the equivalence of the two ratios F/I and U/v for a same vertical position inside the magnetic field is

$$(mg - \Delta\bar{e}g - \Delta\bar{F}_a)/I = \left(\frac{\Delta U_2}{\Delta v_2} + \frac{\Delta U_1}{\Delta v_1}\right) - \left(\frac{\Delta R_2 I}{\Delta v_2} - \frac{\Delta R_1 I}{\Delta v_1}\right). \quad (6)$$

The equation includes two corrective terms as a consequence of the simultaneous scheme which are the force due to the coil acceleration and the resistive voltage drop across the coil due to the current flowing. These are the main difficulties presented by a one-phase watt balance, and will be discussed in the next section.

6. Drawbacks

6.1. Force due to the acceleration

6.1.1. Displacement. In a one-phase watt balance of the BIPM design, the weighing balance measures the total force exerted on the apparatus while the coil moves. It is composed not only of static forces (including gravitational force and electromagnetic) but also the force related to the acceleration a of moving masses. As shown in figure 2(*b*), the weighing balance actually detects the differential acceleration force F_a of the masses placed on the two sides of the driving motor, which move in opposite directions. If we assume the same absolute acceleration value for all the masses in movement, this differential force can be simplified to $\Delta m \cdot a$. The term Δm represents the difference between the masses placed on both sides of the motor. In a symmetric measurement configuration, Δm has the same absolute value $m/2$, m being the mass of the test mass, but of opposite sign in the two measurement steps. The corrective term $\Delta\bar{F}_a$ of (6) could thus be estimated to be

$$\Delta\bar{F}_a = (m/2) \cdot [(a_{1-\mathrm{u}} + a_{1-\mathrm{d}}) + (a_{2-\mathrm{u}} + a_{2-\mathrm{d}})]. \quad (7)$$

Quantities $a_{1-\mathrm{u}}$ and $a_{1-\mathrm{d}}$ are the accelerations of the upward and downward motions when the test mass is removed; $a_{2-\mathrm{u}}$ and $a_{2-\mathrm{d}}$ are those terms when the test mass is loaded. The acceleration of the coil motion can be made very small by careful design of the velocity servo-control loop, including an appropriate driving motor. Moreover, the accelerations of the upward movement, $a_{1-\mathrm{u}}$ and $a_{2-\mathrm{u}}$, and downward movement, $a_{1-\mathrm{d}}$ and $a_{2-\mathrm{d}}$, generally have opposite signs and thus largely cancel out if the coil displacements in the two directions follow exactly the same velocity profile. A small velocity of 0.2 mm s^{-1} is used in the present BIPM watt balance, with a relative variation of 5%. In reality the velocity profile is slightly different between the up and down displacements, probably due to the imperfect servo-control and mechanical imperfections of the flexure strips supporting the levers. Even under these non-optimal conditions, the maximum relative error on the force determination (and thereby on measurement of the Planck constant) is estimated to be about 4×10^{-8}. This error is reduced by an order of magnitude if averaged along the coil travel length. This effect could be larger with an increased velocity but should not pose a problem at the 10^{-8} level. We note that this error is minimized for a one-phase watt balance using the apparatus design shown in figure 2(*a*).

6.1.2. Vibration. In addition to the acceleration due to the imperfect velocity control of the coil motion, the acceleration due to the vibrations introduces noise in the force signal. High-frequency vibrations, even those of small magnitude, can instantaneously generate large accelerations, and thereby introduce significant noise on the force signal. This would increase the type A uncertainty of the F/I ratio.

In practice, the watt balance measures and compares the ratios derived from two experiments, the F/I ratio and the U/v ratio. For the U/v ratio, any vibration noise ε on the velocity $v \cdot (1+\varepsilon)$ will lead to a noise with the same relative effect on the induced voltage $vBL \cdot (1+\varepsilon)$. This common noise in the two signals will be cancelled out by calculating the ratio of the two synchronously measured signals. For the F/I ratio, the current driven through the coil is generated by a highly stable and low-noise current source. The delivered current is only subject to the electrical noise of the regulation loop, which is practically negligible compared with other noise sources. The noise resulting from vibration therefore appears only on the force signal, not on the current signal. Thus any vibrational noise remains after calculating the F/I ratio. In the two-phase measurement scheme, the F/I ratio is statically measured at the central position of the magnetic field; the vibrational noise on the force signal can thus be significantly reduced by averaging it over a relatively long time. In the simultaneous measurement scheme, both the F/I and U/v ratios are continuously and simultaneously measured while the coil moves through the magnetic field. This vibrational noise is not easily characterized due to the complexity of the mechanical assembly and the large number of components. Each fixed or moving component could have its own proper acceleration (magnitude and phase) as a result of vibration. In contrast to the acceleration due to the coil motion, the weighing balance detects a combined noise due to the accelerations in all the components mechanically linked to the weighing balance. This noise needs to be either reduced by minimizing the vibrations or corrected after careful analysis. Vibration isolation is thus of particular importance for a watt balance operating in the simultaneous measurement scheme.

For example, the peak-to-peak noise on the mass reading of the weighing balance is about 0.02 g for a test mass of 100 g when using the existing BIPM apparatus operated under the present environmental conditions. This is due to the large vibration peaks transmitted by the internal support structure and the suspension. The acceleration of the moving coil due to the vibrations reaches a few tenths of $1\,\mathrm{mm\,s^{-2}}$. A reduction by a factor of two is routinely obtained by applying the acceleration correction on the raw force data after a correlation analysis. The relative standard deviation of the force measurements (integration time of 160 ms) obtained during one single up or down measurement is of the order of 10^{-5}. The S/N ratio of the F/I quotient will be improved by a factor of ten when a test mass of 1 kg is used in the future. A further improvement by a factor of five is reasonably expected by reducing the vibration peaks and improving the correlation analysis. The noise due to vibrations on the force measurement should not ultimately be a problem for an apparatus working in the simultaneous scheme.

6.2. Resistive voltage drop

The second drawback of the simultaneous measurement scheme is the resistive voltage drop *RI* across the copper coil. In practice, the voltage drop could be much larger than the induced voltage due to the slow coil motion. Accurate separation of this undesirable voltage from the induced voltage is the main difficulty of a one-phase watt balance. As shown in (6), the error term associated with this unwanted voltage drop is $(\Delta R_2/\Delta v_2 - \Delta R_1/\Delta v_1)\, I^2$. This error relative to the voltage determination can be approximated by assuming the same absolute velocity and induced voltage $|U|$ for all the measurements: $[(R_{2-\mathrm{d}} - R_{2-\mathrm{u}}) - (R_{1-\mathrm{d}} - R_{1-\mathrm{u}})]I/4|U|$. For the same vertical position inside the magnetic field, the error would be zero if the coil resistance is constant when the coil moves upwards and downwards or the change between the upward and downward motions remains the same for the two measurement steps (with and without test mass). The resistance of the copper wires commonly used for the watt balance coil is highly temperature dependent with a thermal coefficient of $\partial R/(R\partial T) \approx 4 \times 10^{-3}\,\mathrm{K^{-1}}$. Therefore, even a small change in coil temperature will lead to a significant error.

In the simultaneous measurement scheme, a constant current is continuously injected through the coil except during the current polarity reversal step. The heating due to the current flow should be very stable between the successive coil displacements and during each measurement step. The environmental temperature drift over time will be the most important perturbation. It should be possible to correct globally by mathematical tools for the smooth, nearly linear temperature drifts experienced. Nevertheless, it is difficult to correct for local and short time variations in the coil temperature. Some of these temperature variations may introduce random errors, which could be reduced by averaging. Other variations could be more or less periodic, leading to systematic errors. For instance, the displacement of the coil, carrying a current, along the magnetic field could dynamically interfere with the environment along the gap and slightly modify its thermal distribution. A relative error below 1×10^{-8} requires a temperature stability better than 0.002 mK between the up and down measurements under the present measurement conditions ($I = 1$ mA, $R = 600\,\Omega$, $U = 0.1$ V). For the data acquired at the central position of the magnetic field where the time interval between the passage in the up and down directions is longest (about 3 min), the corresponding rate of temperature change is about $0.04\,\mathrm{mK\,h^{-1}}$. The time interval is decreased to about 20 s for the positions close to the extremities with a required thermal stability of $0.4\,\mathrm{mK\,h^{-1}}$. This type of thermal variation would probably have the same signature for the two measurement steps and thereby partially cancel out. A relative uncertainty of the order of 10^{-7} on the induced voltage determination was obtained in a measurement campaign carried out using the existing BIPM apparatus conducted in a poor thermal environment [1]. Simple data treatment methods were used to remove the *RI* term in this particular study. Both the global and the local temperature variations are expected to be smaller for a closed magnetic circuit similar to the one which will be used for the improved BIPM apparatus. Its large mass will act as a thermal inertia to reduce any external perturbations and thermal drift. We note that the effect due to the undesirable resistive voltage drop will be ten times greater in the future when working with a mass of 1 kg by injecting a current of 10 mA. Future work needs to be continued to improve sensing of the change in coil resistance by measuring its temperature *in situ* as well as to refine and search for ways to accurately separate it from the induced voltage, for example using a bifilar coil as proposed in [7].

7. Conclusions

The BIPM watt balance has the unique feature of being able to implement both the simultaneous and the two-phase measurement schemes. The validity of the basic principle of a watt balance using the simultaneous measurement scheme has been demonstrated. Although such a watt balance requires additional considerations and imposes specific constraints in the apparatus design and operation, it presents a number of advantages compared with a conventional two-phase watt balance. The measurement scheme is insensitive to changes of the magnetic field and of the coil alignment between measurement phases. Alternating between the one-phase and the two-phase modes will allow one to detect systematic uncertainties specific to each mode. The two main difficulties of the one-phase measurement scheme have been analysed. The force due to accelerations of suspended masses should not be a limiting factor. However, techniques for the separation of the induced voltage and the voltage drop due to the current flow need to be further investigated.

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